Exam in Circuit QED, M2 ICFP
Inductive qubit-resonator coupling

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During the lectures we studied the capacitive coupling between a Transmon qubit and a resonator. In this exercise we will instead analyze the inductive coupling between a Fluxonium qubit and a resonator. Its circuit is displayed in Fig. 1. The goal here is to compute the inductively induced coupling strength, and compare it to the capacitive one. This type of circuit is extensively employed in the field of superconducting qubits [Nature 508, 369–372 (2014), PRB 94, 144507 (2016), PRX 12, 021002 (2022)].

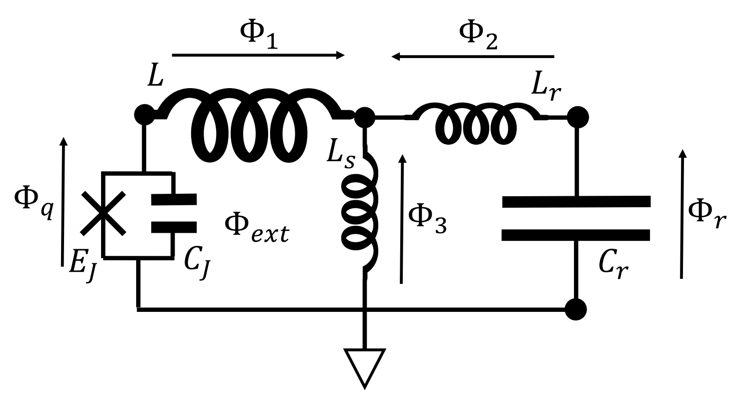

Figure 1: Circuit diagram of a Fluxonium qubit inductively coupled to a resonator.

1. Consider the circuit displayed in Fig. 1. Write down the three constraints that link the degrees of freedom $\Phi_1, \Phi_2, \Phi_3, \Phi_q, \Phi_r$ and the static external flux Φ_{ext}.

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2. Justify that in the limit where $L_r \sim L_s \ll L$, the current conservation law at the central node my be simplified to

$$\Phi_3/L_s + \Phi_2/L_r = 0 \ .$$

3. Express Φ_1, Φ_2, Φ_3 as a function of Φ_r, Φ_q.

4. Compute the potenial energy $U(\Phi_r, \Phi_q)$. Simplify the expression in the limit where $L_r \sim L_s \ll L$. Use the notation $\tilde{L}_r = L_r + L_s$, and the change of variables $\Phi_q \to \Phi_q - \Phi_{\text{ext}}$.

5. Compute the kinetic energy $T(\dot{\Phi}_r, \dot{\Phi}_q)$ of this circuit. Deduce the expression of the charge Q_r, Q_q.

6. Derive the quantum Hamiltonian of this circuit. Comment on the different parts of this Hamiltonian.

7. Express the Hamiltonian as a function of annihilation and creation operators. You may use the notation $\hat{a}, \hat{a}^\dagger$ for the resonator mode, and $\hat{b}, \hat{b}^\dagger$ for the qubit mode.

8. Express the coupling strength g between the qubit and the resonator as a function of L_s and the resonator and qubit resonance frequencies and impedances.

9. Comment on the dependence of g on the resonator impedance, and how it compares to the case of capacitive coupling studied in class. Provide a qualitative drawing of a resonator that favours inductive coupling.

10. (bonus) Provide a strategy to study the *non-linear* coupling between the qubit and resonator induced by the Josephson junction.