Exam in Circuit QED, M2 ICFP
The asymmetric SQUID Transmon

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This exercise is about a widely employed qubit: the asymmetric SQUID Transmon. Its circuit is displayed in Fig. 1, and the goal here is to compute its transition frequency and understand its strengths and weaknesses. An extensive experimental work on this qubit can be found in Ref [Phys. Rev. App. 8, 044003 (2017)]. This type of qubit was used in a recent breakthrough experiment on quantum error correction [Nature volume 605, pages 669–674 (2022)].

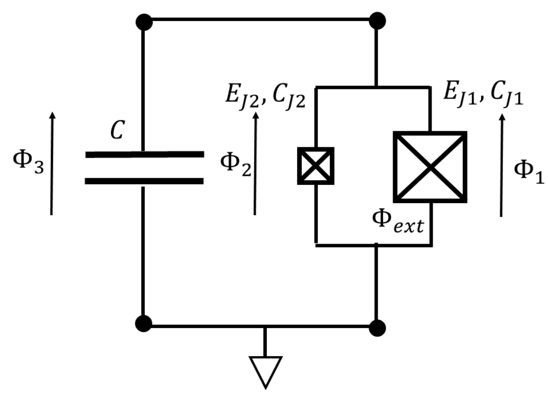

Figure 1: Circuit diagram of an asymmetric SQUID Transmon.

1. Consider the circuit displayed in Fig. 1. Write down the constraints that link the degrees of freedom Φ_1, Φ_2, Φ_3 and the external flux Φ_{ext}. Show that these constraints are fulfilled by the following parametrization

$$\Phi_1 = \Phi + \Phi_{\text{ext}}/2 \quad (1)$$
$$\Phi_2 = \Phi - \Phi_{\text{ext}}/2 \quad (2)$$
$$\Phi_3 = \Phi - \Phi_{\text{ext}}/2 \quad (3)$$

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2. Compute the kinetic energy $T(\dot{\Phi})$ of this circuit. Use the notation $C_\Sigma = C + C_{J1} + C_{J2}$. Deduce the expression of the charge Q.

3. Compute the potential energy $U(\Phi)$ of this circuit. Use the notation $E_{J1} = E_J + \delta E_J$, and $E_{J2} = E_J - \delta E_J$. We will assume that $E_{J2} \ll E_{J1}$ so that δE_J and E_J are both of order $E_{J1}/2$.

4. Show that $U(\Phi)$ may be written as

$$U(\Phi) = -2E_J \cos(\pi\Phi_{\mathrm{ext}}/\Phi_0) f_1(\Phi_{\mathrm{ext}}) \cos(2\pi\left(\Phi - f_2(\Phi_{\mathrm{ext}})\right)/\Phi_0) \ ,$$

where $f_1(\Phi_{\mathrm{ext}})$ and $f_2(\Phi_{\mathrm{ext}})$ are functions of the external flux that must be computed. Give a physical interpretation of $f_2(\Phi_{\mathrm{ext}})$. What is the effective Josephson energy $E_{J,eff}(\Phi_{\mathrm{ext}})$ of this circuit ? Recall that $\cos(\arctan(x)) = 1/\sqrt{1+x^2}$.

5. Derive the quantum Hamiltonian of this circuit. Use the notation $\hat{\varphi} = 2\pi(\hat{\Phi} - f_2(\Phi_{\mathrm{ext}}))/\Phi_0$ and $\hat{N} = \hat{Q}/2e$.

6. We place ourselves in the regime where the Josephson energy is much greater than the charging energy. By analogy to a circuit derived in class, compute the lowest two transition frequencies of this circuit as a function of external flux, and provide a qualitative plot.

7. Give an expression for the dephasing rate of this qubit versus external flux.

8. What are the benefits and drawbacks of this circuit versus the regular Transmon, or the symmetric SQUID Transmon ?